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ABSTRACT

This issue of "Investigations in Mathematics Education" is divided into two sections. Section one contains abstracts and critical analyses for each of six research reports. Two of the reports are concerned with conservation of number, three with instructional procedures, and one with problem solving. Section two is a listing of doctoral theses which were included in "Dissertation Abstracts" from July through December 1969. An instrument for evaluating survey research reports also is included in this issue. (DT)

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Volume 4
November 1971

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INVESTIGATIONS IN MATHEMATICS EDUCATION
A Journal of Abstracts and Annotations

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INVESTIGATIONS IN MATHEMATICS EDUCATION

A Journal of Abstracts and Annotations

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from the Editor

In the past two issues of this Journal we have reproduced materials pertaining to the evaluation of research or research reports.

Volume 2 (July 1969) included Anderson and Kerr's "Checklist for Evaluating Educational Research" and Romberg's "Criteria for Evaluating Educational Research." Volume 3 (January 1970) included Suydam's "Instrument for Evaluating Experimental Research Reports."

Now in Volume 4 we are including Kohr's "Instrument for Evaluating Survey Research Reports." As was true in the case of Suydam's instrument, the evaluation focus is on the research report.

We also wish to call attention to the following two references which pertain to survey research:

C. Alan Riedesel, "Survey Research in Elementary School Mathematics." Arithmetic Teacher 15: 260-263; March 1968.

Thomas A. Romberg, Survey Research: Guidelines for Status Studies. Arithmetic Teacher 15: 639-641; November 1968.

Returning to the matter of experimental research. . .

Your editor has become increasingly concerned with research on mathematics instruction which on the surface may be looked upon favorably in terms of the paradigm involved but which, beneath the surface, reflects some inherent limitations which make implications for instruction tenuous or equivocal at best. We have presented in this Volume a hypothetical--but not atypical--illustration of such an instance which has provoked interesting and enlightening discussion among graduate students and others concerned with research in mathematics education.

The amount of reported research supposedly relevant to mathematics education is increasing greatly, with Piagetian studies leading the list of areas or topics investigated. This situation has prompted second thoughts on the feasibility of this Journal's original intent to have abstracted as much as possible of the published research. There is little question that in the future we shall have to abstract selected studies, and guidelines are being established for this selection process. As will be mentioned shortly, there

are other developments which may prompt further departures from things originally included in the Journal. There is a very real sense in which this belated current issue of Investigations in Mathematics Education is an interim "holding operation" while adjustments to change are being made.

Section 4-1-000 includes just six abstracts--abstracts for which space was not available in Volume 3.

Section 4-2-000 includes a list of 110 doctoral theses and projects which were summarized in Dissertation Abstracts International during the period July-December 1969. An extension of the listing beyond this time would represent a duplication of effort and Journal space, since an annotated listing of "Research on Mathematics Education (K-12) Reported in 1970" [prepared by Suydam and Weaver, which includes references to Dissertation Abstracts International as well as to other journals] will be published in the November 1971 issue of Journal for Research in Mathematics Education. It is intended that this will be a continuing annual feature of JRME, thus obviating the need for any similar listing in Investigations in Mathematics Education. In fact, this JRME listing also raises doubts about the need for a "Listing of Briefly Annotated References" of the kind included in Section 3, Volume 3 of Investigations in Mathematics Education.

Looking ahead. . .

Volume 5 of Investigations is now "in the works" and is scheduled for early 1972 publication. This forthcoming issue will include an expanded section of abstracts of selected journal-reported research. A second section will include an annotated listing of college- and university-level research summarized in Dissertation Abstracts International, since these are not being incorporated in the JRME listing.

INSTRUMENT FOR EVALUATING SURVEY RESEARCH REPORTS

Richard L. Kohr

The Pennsylvania State University

This instrument appears in the following document which is in the public domain:

Marilyn N. Suydam and C. Alan Riedesel, Interpretive Study of Research and Development in Elementary School Mathematics. Volume 2, Compilation of Research Reports. Final Report, USOE Project No. 8-0586. University Park, PA: The Pennsylvania State University, June 30, 1969. Appendix C, pp. 300-303.

[See Kohr's Addenda, pp. 304-311 for an "Explanation of Questions and Key Points," and pp. 312-320 for a "Report of Test Reliability."]

Directions:

The following instrument is to be used for evaluating survey research reports within the framework of curriculum research. It is composed of nine major questions which are underlined. You are to rate the quality of the report in terms of <u>each</u> of these nine questions using the following five-point scale:

(1) Excellent: all requirements for the question are met; nothing essential could be added

(2) Very good: most requirements are met

(3) Good: some requirements are met

(4) Fair: a few requirements are met

(5) Poor: none or too few of the requirements are met

In determining a rating for each question certain "key points" should be considered. These are listed below the question, followed by adjectives which indicate the continuum on which the key point should be assessed. Do NOT make a response to these "key points." They are intended to focus the attention of all raters on the same pertinent aspects of each question. In some studies certain "key points" may be irrelevant. In such cases base your judgment on those "key points" which are relevant. It is also possible that you may think of "key points" not included among those listed under a major question. Where relevant such additional "key points" may be used in assessing that question. There may be some instances in which none of the "key points" seem relevant or where the report fails to supply sufficient information. If this occurs, evaluate the question in terms of what you think should have been done and/or what information should have been included.

Please make only nine responses for each article, one for each question.

1. How practically or theoretically significant is the problem? (1-2-3-4-5)
   a. Purpose (important---non-important)
   b. Problem origin
      (1) Rationale (logical---illogical)
      (2) Previous research (related---unrelated)
   c. Generalizability (extensive---limited)

2. How clearly defined is the survey problem? (1-2-3-4-5)
   a. Objectives and procedures (specified---unspecified) (operational---vague)
   b. Delimitations (noted---not noted)
   c. Variables
      (1) Control (relevant---irrelevant) (operational---vague)
      (2) Dependent (relevant---irrelevant) (operational---vague)

3. How relevant and how well defined is the population? (1-2-3-4-5)
   a. Precise definition of population
      (1) Geographical limits (specified---unspecified)
      (2) Time period covered (specified---unspecified)
      (3) Sociological description (specified---unspecified)
      (4) Sampling units (specified---unspecified)
   b. Relevance of defined population to problem (relevant---irrelevant)

4. How adequate are the sampling procedures? (1-2-3-4-5)
   a. Adequacy of sampling frame
      (1) Time period covered (current---outdated)
      (2) Inclusiveness of defined population (complete---incomplete)
   b. Method of sampling (specified---unspecified) (appropriate---inappropriate)
   c. Obtained sample
      (1) Size (sufficient---insufficient)
      (2) Representativeness (adequate---inadequate)

5. How adequately are sources of error controlled? (1-2-3-4-5)
   a. Sampling error (controlled---uncontrolled)
   b. Non-response (controlled---uncontrolled)
   c. Interviewer bias (controlled---uncontrolled)
   d. Response error (controlled---uncontrolled)
   e. Response set (controlled---uncontrolled)

f. Experimenter bias (controlled---uncontrolled)

g. Teacher effect (controlled---uncontrolled)

h. Control variables (controlled---uncontrolled)

i. Extraneous factors (controlled---uncontrolled)

j. Qualifications of research personnel (interviewers, coders, observers) (experienced---inexperienced) (trained---untrained)

6. How adequate are the measuring instruments? (1-2-3-4-5)

a. Choice of measurement technique(s) (appropriate---inappropriate)

b. Instrument(s)

(1) Development of instrument (pretested---not pretested) (satisfactory---unsatisfactory)

(2) Description of administration and scoring procedures (clear---unclear) (complete---incomplete)

(3) Wording of statements or questions (clear---ambiguous)

(4) Sequence of statements or questions (logical---illogical) (random---fixed)

(5) Evidence of reliability (appropriate---inappropriate) (satisfactory---unsatisfactory)

(6) Evidence of validity (appropriate---inappropriate) (satisfactory---unsatisfactory)

c. Rules for categorizing (specified---unspecified)

7. How appropriate is the statistical analysis of the data? (1-2-3-4-5)

a. Procedures of data collection (specified---unspecified) (careful---careless)

b. Relation of obtained data to objectives (essential---unessential) (sufficient---unsufficient)

c. Descriptive measures

(1) Statistic(s) (appropriate---inappropriate)

(2) Evaluation of descriptive (appropriate---inappropriate)

(3) Establishment of relationships (appropriate---inappropriate)

d. Statistical tests

(1) Basic assumptions (satisfied---unsatisfied)

(2) Relation to procedures (appropriate---inappropriate)

(3) Significance levels (specified---unspecified)

e. Description of results (accurate---inaccurate)

8. How reasonable are the conclusions drawn from the data? (1-2-3-4-5)

a. Interpretations (consistent --inconsistent)
(reasonable---exaggerated)

b. Generalizations (reasonable---exaggerated)

c. Implications (reasonable---exaggerated)

d. Qualifications

(1) Discussion of methodological problems and errors (comprehensive---limited)

(2) Alternative explanations (noted---not noted)

(3) Other limitations (noted---not noted)

9. How adequately is the research reported? (1-2-3-4-5)

a. Organization (excellent---poor)

b. Style (clear---vague)

c. Grammar and mechanics (excellent---poor)

d. Completeness (excellent---poor)
(replicable---unreplicable)

e. Presentation of statistics (complete---incomplete)
(clear---unclear)

Implications for Instruction: ?

A hypothetical investigation. . .

Many studies are undertaken in the hope that findings and conclusions will lead to specific implications for instruction. It is not uncommon, however, to read research reports in which the design of an investigation makes any such implications necessarily tenuous at best. Let us consider a hypothetical instance which is far from atypical.

We shall assume that elementary-school pupils have had prior work intended to help them develop the following two abilities, stated in the vernacular of "behavioral objectives":

1. Given a fractional number named by a fraction $\frac{a}{b}$, the pupil renames the number as a fraction $\frac{j}{n}$, where n is some multiple of b.[1]

2. Given two distinct fractions $\frac{a}{b}$ and $\frac{c}{d}$, the pupil specifies whether or not the two fractions name the same fractional number. (When doing this, the pupil applies the preceding objective: he renames $\frac{a}{b}$ as $\frac{j}{n}$, he renames $\frac{c}{d}$ as $\frac{k}{n}$, he recognizes whether or not $j = k$, and he infers from that whether or not $\frac{a}{b} = \frac{c}{d}$.)

Now our hypothetical investigator wishes to compare two seemingly different mathematical approaches to an attainment of the following objective, which is an extension of 2:

3. Given two distinct fractions $\frac{a}{b}$ and $\frac{c}{d}$, the pupil specifies: (i) whether or not the two fractions name the same fractional number; and if not, (ii) which fraction names the greater number and which fraction names the lesser number.

For our purposes here, the two mathematical approaches to be considered may be characterized in the following abbreviated ways:

Treatment X. Rename the fractions $\frac{a}{b}$ and $\frac{c}{d}$ as fractions having the same denominator; $\frac{j}{n}$ and $\frac{k}{n}$, respectively. One and only one of these things will be true (Trichotomy Property):

---

[1] As used here, a fractional number is a non-negative rational number. A fraction is a name for a fractional number: a symbol of the form $\frac{a}{b}$, where a and b designate whole numbers (elements of $W = \{0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$) such that $b \neq 0$.

$$j = k, \text{ so } \frac{j}{n} = \frac{k}{n} \text{ and } \frac{a}{b} = \frac{c}{d}, \text{ or}$$

$$j > k, \text{ so } \frac{j}{n} > \frac{k}{n} \text{ and } \frac{a}{b} > \frac{c}{d}, \text{ or}$$

$$j < k, \text{ so } \frac{j}{n} < \frac{k}{n} \text{ and } \frac{a}{b} < \frac{c}{d}.$$

Treatment Y. Without renaming either fraction $\frac{a}{b}$ or $\frac{c}{d}$, compute the "cross products" a x d and b x c, or simply ad and bc. One and only one of these things will be true (Trichotomy Property):

$$ad = bc, \text{ so } \frac{a}{b} = \frac{c}{d}, \text{ or}$$

$$ad > bc, \text{ so } \frac{a}{b} > \frac{c}{d}, \text{ or}$$

$$ad < bc, \text{ so } \frac{a}{b} < \frac{c}{d}.$$

Our intrepid hypothetical investigator conducts his investigation in accord with the paradigm outlined in Figure 1, where Treatment Z is a placebo treatment (intended to help pupils distinguish squares from other rectangles) and where the interpolated work of Stage 4 carefully avoids anything even remotely associated with Treatments X, Y, and Z.

Let us assume that our investigator made appropriate random assignments of pupils and teachers to treatments; that his sample sizes were wholly adequate; that his treatments were translated into prescribed instructional materials and procedures which were followed and used as intended; that his premeasure(s), criterion measure(s), and retention measure(s) were satisfactorily reliable and valid with respect to the objectives identified earlier; and that he used appropriate sampling units when analyzing his data.

Stage	Activity		
1	Premeasure(s)		
2	Treatment X (40 minutes)	Treatment Y (40 minutes)	Treatment Z (40 minutes)
3	Criterion measure(s)		
4	Interpolated work (5 weeks)		
5	Retention measure(s)		

Figure 1. Experimental paradigm

Our hypothetical investigator identified two principal hypotheses which he elected to test using one-way ANOVAs with predetermined rejection levels, first having ascertained that on the premeasure(s) there were no statistically significant ($p > .05$) differences among treatment means ($\bar{X}$, $\bar{Y}$, $\bar{Z}$):

Stage 3. H_0: $\mu_X = \mu_Y = \mu_Z$.

Stage 5. H_0: $\mu_X = \mu_Y = \mu_Z$.

If either hypothesis were rejected, subsequent tests would be used by the investigator to identify the treatment(s) responsible for rejection.

Now. . . regardless of the investigator's findings, we can be certain of at least one thing: implications for instruction must be tenuous at best. Guidelines for teachers must be highly equivocal, if suggested at all.

What theoretical and practical considerations, and what experimental uncertainties, account for such a pessimistic point of view toward this hypothetical investigation? What might have been done to increase the likelihood that the research findings--whatever they may be--would warrant less tenuous implications for instruction, less equivocal guidelines or suggestions for teachers?

Section 4-1-000

Abstracts of Research Reports

4-1-001. Rothenberg, Barbara B. "Conservation of Number Among Four- and Five-Year-Old Children: Some Methodological Considerations." Child Development. 40: 383-406; June 1969.

1. Purpose

The major purposes of Rothenberg's study were

(a) to investigate conservation of number using prior assessment of key terms and a question format by which she attempted to decrease problems of complexity and bias present in previous studies, and

(b) to study the effect on conserving status of different types and number of transformations.

Of lesser concern were

(1) the need for including justification of responses in assessing conservation status when using the standard question format,

(2) possible differences in conservation status when subjects (Ss) were asked to solve problems of conservation of equality and inequality,

(3) the effect of materials on conservation status, and

(4) relationships of conservation of number with each of intelligence, age, social class, and sex of the Ss.

2. Rationale

Standard question formats have been utilized to measure conservation attainment since they provide a more comparable situation for all Ss than does Piaget's more flexible clinical method of questioning. Major difficulties within the limits of standard formats are in the vocabulary level and structure of the questions. If a S fails to conserve, it has not been possible to know whether this failure was due to inability to understand the language of the question, the concept of conservation, or both. The difficulties arising from the structure of the question are seen in the format of the commonly posed conservation question, "Does this row (or side) have more, or does this row (or side) have more, or do they both have the same number (amount, etc.)?" Such questions are difficult for young children to remember as well as being difficult to answer with a single response.

Varying criteria have been used to define conservation. A fundamental difference in such criteria resides in whether or not the Ss are asked to explain their response(s) to the conservation question. Changes in the nature of the conservation questions to provide a larger sample of responses

may decrease discrepancies in results found when requiring explanations or not.

Another aspect of methodology which required further consideration is the nature of the transformation used to measure conservation of number. Previous studies have limited the transformations to only two types, expanding and collapsing. Use of additional types of transformations should show how generalizable the results are from the more typical transformations.

It has been found that provoked correspondence, using functionally related materials, tends to facilitate conservation of number to a greater extent than unprovoked or spontaneous correspondence.

3. Research Design and Procedure

The subjects were 210 lower and middle class preschool and kindergarten children ranging in age from 4 years 3 months to 5 years, and 5 years 3 months to 6 years, respectively. They were individually tested by trained examiners in a separate room at their school.

The conservation task was developed by incorporating four modifications into the procedures and materials of past measures of number conservation. The modifications were:

(1) an 18 × 24 inch board was colored half yellow and half blue so that an array placed on one color would be clearly distinguishable from an array placed on the other color,

(2) assessment of and instruction in concepts basic to an adequate performance on the tasks,

(3) the use of a warm-up item, and

(4) a modification of the wording of the conservation question to provide single section questions. Five types of transformations of objects were used along with two materials sets. A description of a typical item is as follows (all questions are by E, after a transformation):

1. Does this bunch have the same number of blocks as this bunch? Whether "yes" or "no" go on.

2. Does one bunch have more blocks? If "yes":

   a. Which bunch?
   b. How can you tell?
   c. Change the blue side so it has the same number of blocks as the yellow side.

If 1 is "no" and 2 is "no": Ask question 1 again. If "yes" go on to the next transformation. If "no," say:

   a. Which bunch has more blocks?
   b. How can you tell?
   c. Change the blue side so it has the same number of blocks as the yellow side.

If 1 is "yes" and 2 is "no": How can you tell?

For a child to be classified as a consistent conserver and obtain a score of 4, he had to respond "yes" to question 1, "no" to question 2, and understand the nature of the language. For a child to be classified as a consistent nonconserver and obtain a score of 2, he had to respond "no" to question 1, "yes" to question 2, and understand the nature of the language. All other responses were given a score of 0, and children who obtained them were called inconsistent nonconservers. The responses to the questions "How can you tell?" were assigned to one of seven categories. A verbal comprehension measure, the Peabody Picture Vocabulary Test (PPVT), was administered to all Ss.

4. Findings

(1) The effects of materials (provoked and spontaneous correspondence) was nonsignificant for each χ^2 analysis performed for each transformation. Consequently, all other findings are presented for all children combined.

(2) A warm-up item was administered which did not entail an understanding of conservation in its solution. It provided base-line data on the Ss' understanding of "more" and "same." For the sample as a whole, only 58 percent seemed to understand the necessary language. The lower-class 4-year-olds had a very small percentage of correct responses on the item. The lower-class subjects showed more deficiency in their basic understanding of the above terms than middle-class children ($z = 6.75$, $p < .001$).

(3) Nearly twice as many conservers were identified when only their accurate response to the question of "same" was required than when correct responses to both questions "same" and "more" were required ($z = 2.72$, $p < .001$).

(4) Most of the children who gave conserving responses were not consistent in giving such responses for each transformation. Ninety two children conserved on one or more transformations, 45 on two or more, 31 on three or more, 19 on four or more, and 13 on all five. Therefore, true conservation status of a child appears not to be reliably determined on the basis of one or even two types of transformations.

(5) The explanations providing justifications for responses were reclassified "adequate" and "inadequate" responses. Ss with conserving responses had 59 percent adequate explanations, as compared with 34 percent adequate reasons for the consistent nonconservers and 19 percent for the inconsistent nonconservers. Among Ss who gave conserving responses, 39 percent of the lower-class group had good explanations, as compared with 64 percent of the middle-class children. Nonconserving Ss depended particularly on the perceptual category in their justifications.

(6) Since there were five transformations and a possible score of 4 on each transformation, the range for total conservation scores was 0-20, not considering justifications.

The results of a 2 × 2 ANOVA among lower-class Ss ($N = 77$) showed a significant difference ($p = .02$) due to age but not sex. Among middle-class Ss, no significant main effects for age or sex occured. For the total sample ($N = 183$), there was a social-class by age interaction ($p < .05$) which showed on age increase for lower-class Ss but not middle-class Ss.

(7) The correlation between PPVT raw score and conservation of number total score was .52 ($p < .001$).

5. Interpretation

Rothenberg concluded that:

(1) "The understanding of key words 'same' and 'more' was. . . less than complete. . . particularly among the lower-class children. . ."

(2) "nonconserving children. . . make. . . irrelevant interpretations of the words 'same' and 'more' when these are used in conservation of number problems."

(3) "about half as many conservers were identified on each transformation when correct responses to questions of 'same' and 'more' were required. . . than when just one correct answer was necessary."

(4) "Among equality items. . . there were approximately the same percentages of conservers on all five types of transformations. . . ."

(5) "a particular subject should probably not be considered a conserver of number unless he is able to demonstrate his conserving ability through a variety of different problems."

(6) "The obtaining of explanations from young children will be less than complete because of low verbal ability of many children."

(7) Future investigation of. . . conservation. . . might profit from having increased precision and reliability in their measuring instruments.

Abstractor's Notes

Those researchers interested in assessing mathematical learning of young children would benefit from reading Rothenberg's work. She has contributed to the solution of thorny methodological problems when assessing "conservation of number." For example, researchers should now be explicitly aware of the dangers of misclassification of "conservers" if they utilize only one or two items on which to base that classification. Moreover, researchers should not feel compelled to obtain verbal justifications from the children if they use single-section questions with a variable "yes" or "no" response format.

There are problems which Rothenberg did not consider. While they do not necessarily detract from her study, they should be mentioned. First, Rothenberg may not have been studying conservation of number at all, but instead conservation of numerical-type relations. Surely cardinal numbers must be distinguished from relations defined on those numbers. Such terminology as "same number" and "more than" are relational in nature, and should be conceived of as such. Such terms may refer to relations between sets or between numbers, depending on stimulus situations and on the child himself. Second, if a child establishes a relation of equivalence between two collections of objects by setting up a one-to-one correspondence between those collections and then is asked to question which entails knowledge if an order

relation, it may be that the child at that point is being asked to utilize logical interrelationships among relational types (order and equivalence). In any case, such logical interrelationships need to be considered when assessing "conservation of number" and any other forms of "conservation."

Leslie P. Steffe
University of Georgia

4-1-002. Wallace, John G. "An Approach to the Acceleration of Number Conservation in Children." Educational Review 20: 224-238; June, 1968.

1. Purpose

To determine the possibility of accelerating the development of number conservation when conservation is measured by verbal and non-verbal tests which demand a degree of generalization sufficient to detect rule learning.

2. Rationale

There are published studies available designed to assess the feasibility of accelerating conservation of number using various training methods. (See for example, Churchill, E. M. "The Number Concepts of the Young Child" Leeds University Research and Studies. 17, 34-49, and Wohlwill, J. F. and Lowe, R.C. "Experimental Analysis of the Development of the Conservation of Number" Child Development, 33) There is reason to believe, however, that the methods of training used were either too general to assess the effect of specific types of experience (Churchill) or too restricted because the child may have developed an empirical rule for responding to a test item. (Wohlwill and Lowe) The present study was designed to avoid the restrictions mentioned above.

3. Research Design and Procedure

The 227 children employed in this study were divided into three age groups. Thirty-seven between 4.0 and 4.6 years; ninety-seven between 5.0 and 5.6 years; and ninety-three between 6.0 and 6.6 years. Both the control and the experimental groups were given a non-verbal test (Wohlwill's) and a verbal test (Dudwell's). Both groups were given the above tests as a pre-test and a post-test of number conservation. The experimental group was composed of 90 children selected from the original population of

227 children. These 90 children were selected on the basis of three types of non-conservation performance detected by the pretests.

The type of training procedure used for the experimental group was determined on the basis of two hypothesis:

(1) "Exposure to a mixed sequence of addition-subtraction and conservation trials will accelerate attainment of number conservation." (This is in contrast to the somewhat limited variation of training experiences used in other studies.)

(2) "True conservation, as distinct from the requisition of a narrow, empirical rule, will be produced by confronting children with an extended series of relevant learning experiences involving the use of a wide variety of materials."

At the end of an eight day training session (30 minutes per day), the subjects in the experimental group and control group were given the verbal and non-verbal test (post-test). Three months later, the subjects were retested using both tests employed as post-tests.

4. Findings

To interpret the results of the testing procedure, it was necessary to establish an arbitrary "pass" point. On the basis of this "pass" point, the number of children from the experimental group who passed the verbal test exceeded those in the control group by 2 to 1 ($\frac{15}{8}$) (There were 30 children in each group at the start of the training session). On the second post test, this ratio was $\frac{23}{14}$. On the non-verbal test these ratios were $\frac{7}{4}$ and $\frac{16}{3}$ respectively.

5. Interpretation

The investigators conclude

(1) "The effectiveness of the training procedure is clearly indicated by the results of the Experimental and Control Groups on the second post test."

(2) ". . . the findings appear to support the conclusion that the two main procedures on which the training method was based may well constitute not only sufficient, but also necessary conditions."

(3) "They (the results) suggest that the perceptually dominated approach, which is characteristic of non-conservation and stems from the dominancy of the deformation schema, does not come into conflict with the addition-subtraction schema but is abandoned before the addition-subtraction schema becomes of central importance. It is abandoned when the child becomes aware of the additive composition of conservation and is brought about by the child's attainment of the ability to count."

(4) "A tentative account of this development of number conservation is offered in which the ability to count and experience of juxtaposed addition-subtraction and conservation are accorded important roles."

Abstractor's Notes

Status studies of this kind are important. What effect does ability to conserve have on a child's success or non-success with the basic ideas and skills of arithmetic?

Throughout this study and many others whose focus is on conservation of number there is a hazy, if not a complete, misconception of the relation between counting, one-to-one correspondence, and the cardinal number. Adults and children can answer questions involving "as many as," "more than," etc., without the ability to count. And there exist children who can count objects up to ten with great accuracy but will not admit that two counted sets of 8 objects each have the same number of objects. Furthermore, asking 4 year old children "Are there the same number of objects now?" is equivalent to asking them about the existance of polar bears on the equator. From personal experience, the writer has found children who stare at you or point seemingly at random when one places candies as below and asks, "Are there the same number of candies in each row?" (or, pointing, here and here). On the other

Row A O O O O
Row B O O O O

hand when one tells them "You may have this row of candies or this row. Which one do you want?" they look at you in surprise and say, "no difference."

The behavior one expects from a child who has a number concept is never specified by experimenters. Furthermore, the structure of ideas such as one-to-one correspondence, number, transivity, etc., is not exhibited. Sometimes some experimenters will realize that specifying the structure of the idea he is working with is essential for interpreting the data of an experiment.

The present study does not describe the basic population of 227 children. What generality does an experiment based on 227 children have? On the other hand, this experiment does demonstrate the feasibility of a particular method for accelerating the conservation of number. However, the method may not be necessary and sufficient as the investigator claims.

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4-1-003. Brownell, William A. "Conceptual Maturity in Arithmetic Under Differing Systems of Instruction." Elementary School Journal 69: 151-63; December, 1968.

1. Purpose

The major purpose of the investigation was to determine the comparative success of the Conventional, Cuisenaire and Dienes programs of instruction in promoting the movement toward meaningful and functional mathematical abstractions. Three subordinate purposes were

(1) to study the relationship between progress in conceptual maturity and progress in proficiency;
(2) to determine the influence that might be exerted on the effectiveness of the programs by differences in amount of instruction, and in pupil ability; and
(3) to identify special strengths and weaknesses of each of the programs in fostering progress toward conceptual maturity.

2. Rationale

The investigator states that it has been customary to assess the effectiveness of a given arithmetic program and to compare relative effectiveness of different programs by using paper-and-pencil test results. Such data usually represent scores of skill in computation and in problem solving as identified by the test. Although such data provide information relative to achievement, they indicate little or nothing about the maturity underlying thought processes and the amount of understanding attained.

3. Research Design and Procedure

Forty-five schools in Scotland and England were selected as the basis for the population of this study, subdivided into two studies--Scottish and English. Those students completing their third year of schooling and who had been exposed to only one of the three programs--Conventional, Cuisenaire, or Dieness--made up the sample for each study. Although these programs are not completely distinct from each other, unique characteristics can be identified. Instruction in the Conventional program was based on counting. The Cuisenaire program was based on the well-known Cuisenaire rods which stress relationships rather than an early mastery of number facts and computation. The Dienes program was based on the Multibase Arithmetic Blocks ("units," "longs," "flats," and "blocks") intended to introduce children to the principle of numeration.

The distribution of the schools and subjects is presented in Table I.

Table I

Programs

	Conventional	Cuisinaire	Dienes
Scottish study			
Schools	6	7	
Subjects	253	225	
English study			
Schools	12	10	10
Subjects	313	257	358

Three sets of tasks were used to ascertain the effectiveness of each of the programs for basic facts, computation, and problem solving. There were eight combinations (two for each of the four arithmetic operations), four computations (one for each of the four arithmetic operations) and twelve one-step verbal problems.

Each child was observed as he worked and was questioned after each attempt by a trained interviewer. Responses observed, volunteered, or elicited by questions were classified on a coded blank on the basis of thought process, proficiency (correctness of response) and performance (manner and time in

which task was performed). Other data recorded were age (in 6 months brackets), sex, brightness*, achievement in arithmetic* and reading achievement.*

Gross comparisons were made between two total programs (Conventional and Cuisenaire) in Scotland, among three program groups in England, and between high and low subgroups within a program. Differences were arbitrarily regarded as significant only if they amounted to at least 10 percent. Differences of 5 percent or less were regarded as insignificant and differences of from 5 to 9 percent were viewed as being of doubtful significance.

4. Findings

Scottish Study

(1) Children in Scottish schools who used the Cuisenaire program demonstrated much greater maturity of thought processes in finding answers for the number combinations and much more ability to explain the mathematical rationale of computation than the subjects of the Conventional program.

(2) There was no significant difference between the two programs (Conventional and Cuisenaire) in verbalizing reasons for methods of solving verbal problems.

(3) There was no difference between the two programs in proficiency (accuracy of answers). It is noted, however, that those using the latter had twenty percent less instructional time.

English Study

(1) The Conventional program had the highest over-all ranking for effectiveness in promoting conceptual maturity. The Dienes and the Cuisenaire programs ranked about equal to each other in this respect.

(2) The Conventional program ranked first on maturity of thought processes on basic facts, the Cuisenaire program second, and the Dienes program third. Difference between the latter two programs was slight, however.

(3) The Conventional program ranked first on maturity of thought on computation, and the Cuisenaire and the Dienes programs ranked about the same.

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*Determined by teachers' estimates. No standardized test was used.

(4) The Conventional, Cuisenaire and Dienes programs were about the same on maturity of thought on problem solving.

(5) The group with the higher rank in conceptual maturity was also higher rank on proficiency for both combinations and computation.

(6) For comparison between high and low ability groups within a program, the Cuisenaire program appeared to be more effective than the Conventional program per unit of time. For less instructional time, the programs were effect in the order: Conventional and Dienes; Cuisenaire.

(7) The Cuisenaire program was more effective than the Conventional program in teaching children rated high in brightness to move steadily to higher level of thought processes and mathematical understanding in all three arithmetical tasks.

(8) For children at the lower end of the brightness scale, the rating was as presented in Table II.

Table II

Ranking For Low Brightness Groups

Program	Tasks: Combinations	Computation	Problem Solving
Conventional	1st	1st	2nd
Cuisenaire	3rd	2nd	3rd
Dienes	3rd	2nd	1st

5. Interpretation

(1) There was reason to believe that the Cuisenaire program was better taught than the Conventional program in the Scottish schools and that the Conventional program was better taught than the Cuisenaire program in the English schools.

(2) Programs are paper orgnizations. The quality of teaching of the two programs in the two countries should not be overlooked in determining the relative effectiveness of the programs.

(3) Differences between the programs may also be attributed to too short a time on a variety of systems of notation before encounter with the decimal system. The result was superficial ideas that had no function in the second and third years of school and that had no tie to other learning.

Abstractor's Notes

This report represents a needed effort for assessing the outcomes of instructional materials and the use of those materials upon children. To measure only one aspect of learning in terms of proficiency provides a limited picture of a student's development in conceptualization, mathematical understandings, skills, and the use of those competences in solving problems.

The investigator carefully noted limitations and the many uncontrolled variables and perhaps variables that cannot be controlled in educational research. To recognize these limitations in educational research is necessary, if findings are to be used to effect change in curriculum and instructional procedures. Furthermore, identifying contributable characteristics of each of these programs to the improvement of mathematics programs which no one of the three might otherwise accomplish alone is noteworthy.

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4-1-004. Suppes, Patrick, Loftus, Elizabeth F., and Jerman, Max. Problem Solving on a Computer-Based Teletype. Institute for Mathematical Studies in the Social Sciences, Technical Report No. 141. Stanford California: Stanford University, March, 1969, page 22.

1. Purpose

The purpose of the paper was to determine factors affecting the difficulty of word problems and to formulate and test some linear structural models using these factors that lead to prediction to relative difficulty of word problems. This study is the first Stanford study on word problems, but is one of a series of studies formulating process models for various arithmetic computations.

2. Rationale

The models developed are based upon an attempt to find objectively identifiable factors in the problems themselves upon which to base a prediction of the relative difficulty of the word problems. The literature and previous research yielded two types of variables, "0,1" variables and multivalued variables. Of the latter type, three variables were chosen, the minimum number of different operations for solution, the minimum number of

steps needed for solution, and the word length of the problem. This latter variable was chosen on the basis of a generalization from the research on sentence processing in young children. There were three "0,1" variables used, a sequential variable (whether the immediately preceding problem was solvable by the same steps in the same order), a verbal clue variable (did the problem contain all of the needed verbal cues to indicate the operations to be used) and a conversion variable (was a conversion of units required).

Use of the linear regression model allows for the assignment of weights to each of this small collection of objective variables in order to predict the relative difficulty of a large number of word problems. This is the reason for the choice of this type of model in this setting.

3. Research Design and Procedure

Twenty-seven bright fifth-grade students, with experience in using teletype terminals for drill and practice in arithmetic, were taught how to develop their own simple computational procedures for solving word problems using the computer via a teletype.

After this instruction was given, the students solved 68 word problems, all having numerical answers, judged to be of sixth grade difficulty. These items had been studied for presentational and procedural difficulties in a small pilot study. In the study the computer presented the problem and then a list of all of the numbers in the problem. The student, without aid of pencil and paper and using rules taught him in the instructional period, developed a step-by-step computational procedure to solve the problem. Any valid set of steps was accepted by the computer.

The computer checked the student-indicated answer and gave the student feedback as to its correctness and then presented the next problem. All students attempted all 68 problems, apparently in a sequence of sessions of approximately 6 minutes each. Each item was scored as being right or wrong for each student.

4. Results

Using a computer procedure for multiple linear regression a regression equation was obtained:

$$Z_i = -7.36 + .87X_{i1} + .18X_{i2} + .02X_{i3} + 2.13X_{i4} + .26X_{i5} + 1.42X_{i6}$$

where $Z_i = \log\left(\frac{1 - p_i}{p_i}\right)$, where p_i is the proportion of students getting

item 1 correct. This transformation was used to preserve probability with respect to predicted proportions. There were three significant variables, sequential, conversion and number of operations, with the first of these apparently the most important. The multiple R for this equation was .67; thus the model accounted for about 45 percent of the variance in probability of a correct response. A χ^2 measure of fit yielded $\chi^2 = 555.76$, indicating a highly significant difference between predicted and observed proportions correct on these items.

A regression equation based on 63 problems (removing the problems accounting for the most variability between predicted and observed values) yielded a multiple R of .73 and a reduced, but still significant χ^2 measure of fit. An equation using only the variables of number of operations, number of steps, sequential and conversion yielded one only slightly reduced multiple R on the reduced set of 63 problems. These latter two analyses on the reduced set were done for exploratory reasons and are not considered valid for inferential purposes.

5. Interpretations

(1) Although the results are crude, they represent a first step toward ordering a complex set of problems using only a few variables.

(2) It is difficult to build a processing model that accounts for all of the difficulties students encounter in solving word problems.

(3) A full behaviorally sensitive syntactic and semantic analysis of word problems is needed to predict all the details that must be accounted for.

Abstractor's Notes

(1) Since a small number of variables is almost a necessity in using linear regression methodology, (or else an intolerably large number of test items) choice of these variables in explaining more of the variance is paramont. In this study the sequential variable was most significant; would not variables related to the work of Polya like "remoteness of a problem like the problem at hand" be useful? Could Polya's strategy of breaking a problem down into simpler problems, be quantified and used in a sequential sense?

(2) The model developed seemed very good for approximately the 40 easiest items, but poor for the more difficult items. Is the linear model appropriate, or is some other model needed?

(3) In what sense did variability in the students' histories with word problem solving affect the results here? Would the results differ with a different student population?

(4) As part of the research procedure a record of each student's computer algorithm for each of 68 problems was collected. The abstractor is fascinated by the possibility of studying these algorithms as a means of finding sources of problem difficulties both in terms of the problems and the process. I hope that reports on a study of this data as well as continued attempts in refining our theoretical knowledge are forthcoming.

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4-1-005. Greabell, Leon. "The Effect of Three Different Methods of Implementation of Mathematics Programs on Children's Achievement in Mathematics." The Arithmetic Teacher, 16: 288-292; April 1969.

1. Purpose

To determine the relative effectiveness of three approaches to implementing elementary mathematics programs in fifth grade. These approaches were categorized as

(1) Systematic Modern,
(2) Crash Modern,
(3) Traditional.

2. Rationale

The approaches to implementation were categorized according to

(1) amount of systematic study of programs before selection,
(2) amount of in-service training of staff, and
(3) the basal text series being used in the system.

Cited sources have stated that an in-service program is necessary for successful implementation. Cited studies have indicated that the chief

difficulty in determining the effectiveness of mathematics program has been the weakness of standardized tests in measuring concepts.

3. Research Design and Procedure

Three different instruments were selected to represent the characteristics of the three approaches:

(1) The Iowa Test of Basic Skills, Form I (1956), Houghton Mifflin Company. This was selected for measuring skills emphasized in traditional programs.

(2) Stanford Achievement Test, Intermediate II, Form " (1964), Harcourt, Brace, and World. This was selected because it measures both traditional and modern characteristics.

(3) Contemporary Mathematics Test, Upper Elementary Form " (1966), California Test Bureau. This was selected to measure modern concepts and skills.

Three school systems were chosen to represent the three categories of implementation. Seventy-five children were randomly selected from the fifth grades of these systems, twenty-five for each test. The number of boys and girls were about the same. In the traditional system the mean I. Q. (Lorge-Thorndike) for Grade 5 was 111.4. In the crash modern system the mean I. Q. (Lorge-Thorndike) Grade 5 was 109.6. In the systematic modern system the mean I. Q. (California Test of Mental Maturity) for Grade 5 was 113.1. No significant differences among I. Q. scores were found for the sample group. Consequently the differences in the achievement scores (grade equivalents) were tested by analysis of variance.

4. Findings

There were significant differences ($p < .01$) between implementation groups on the computation and concepts parts of the Stanford Achievement Test. There were no significant differences on the other test scores. The systematic modern system had the highest mean on the Contemporary Test and the Stanford Test. The crash modern system had the lowest mean on the Iowa and Stanford Test. The traditional system had the highest mean on the Iowa Test.

5. Interpretations

The author states: "Are the differences inherent in the traditional curriculum and the modern curriculum different enough so that they can be

measured by a standardized instrument, or can any practical group instrument be devised that will measure the differences? If these differences are non-measurable, how can we be sure that the goals of a modern curriculum are being attained?"

Abstractor's Notes

It is not clear whether the study was designed to investigate the effectiveness of implementation programs or the effectiveness of measurement instruments in evaluating programs. The report of the investigation was not clearly expressed.

The selection of the sample was quite inadequate to measure the effectiveness of implementation methods. Why were fifth grade children selected to use as the sample? The length of time the systems had been under a change of program is significant, but not discussed.

The dismissal of I. Q. differences is not adequately explained. Were the two different I. Q. measures equated? Were the differences in I. Q. compared across implementation groups or across the test groups?

The table of mean grade equivalents and the analysis of variance results do not seem to fit. E.g., the means for the Stanford concepts subtest were 6.47, 6.89, 6.47 and yielded an F-ratio of 19.0, while the application subtest had means of 5.54, 6.67, 6.48 and yielded an F-ratio of 3.05. The total mean grade equivalents do not seem to fit either. Could there be an error in the tables as printed?

Even though this was termed a "pilot study" the intent and design of the investigation were not discussed thoroughly enough.

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4-1-006. Pigge, Fred, and Brune, Irvin H. "Lectures Versus Manuals in the Education of Elementary Teachers." Arithmetic Teacher 16: 48-52; January, 1969.

1. Purpose

"The main purpose of this study was to determine whether the individual study of intermediate grade teachers' manuals would enhance a group of pre-service teachers' understandings of several basic mathematical concepts.

"A secondary purpose of the study was to determine which of two methods--(1) the above described individual study of the teachers' manuals and (2) a lecture approach--would be associated with a greater mean gain on a test which purported to measure the attainment of basic mathematical understandings."

2. Rationale

Even with the increased requirement of courses in mathematics content for elementary education students, instructors in mathematics education ". . . recognize that these students need a new look, if not a thorough review, into some key ideas." Typically the approach to the solution of an instructional need is the provision of lectures. Is an approach ". . . more consonant with the situation in which future elementary teachers will work" useful in mathematics education courses? Will the use of the improved teachers' manuals facilitate this problem?

3. Research Design and Procedure

"Three college classes in elementary mathematics methods were randomly assigned to each approach; therefore, a total of six classes were involved in the study." Two instructors were involved in the treatments: one taught two lecture and two textbook groups; and the other taught one lecture and one textbook group. The three topics included during the three week experimental period included:

(1) place value, structure, and algorithms for whole numbers;
(2) geometry and measurement; and
(3) fractions.

Treatment A: Students were provided copies of 4th, 5th and 6th grade manuals for modern arithmetic series. The manuals included text for teachers on the same page with a copy of the pupil page. Students studied the manuals for about thirty-five to forty minutes each period and spent the last ten to fifteen minutes discussing what the authors were attempting to do with the content topic at a given grade level.

Treatment B: Students were taught strictly by a lecture approach. The instructor planned his lecture to cover the same general content as found in the teachers' manuals 'especially in the Mathematical Background sections' on each topic.

A pretest was administered to both groups and at the end of the nine lessons a second form of the same test was administered as a post-test. The criterion measure was the mean gain score between pretest and post-test. The fifty-eight items were distributed among topics as follows:

(1) twenty-two items: place value, structural properties, and various algorithms for whole numbers;

(2) fifteen items: geometry and measurement concepts; and

(3) fourteen items: fractional understandings.

A correlation of .90 was obtained as a reliability measure when the two forms of the test were administered to sixty-six pre-service teachers not otherwise involved in the study. The mean scores of pretest and post-test were essentially alike indicating that taking the first did not appear to result in higher scores on the second.

4. Findings

Both treatment groups made significant gains in their scores on the post-test and neither made significantly greater gain than the other.

5. Interpretation

"It is the authors' opinion that students who studied the teachers' manuals probably gained more than the test measures, and perhaps more than the lecture group had a chance to gain. The rationale behind such a statement is that the teachers' manual group was exposed to the 'tools of their future trade,' namely, pupils' textbooks and teachers' manuals. This group of students became quite familiar with the structure of the teachers' manuals, with the content, and with methods of presenting the content at the three grade levels. They should therefore feel more secure and competent in teachers elementary arithmetic because of that exposure. This conjecture, i.e., that studying teachers' manuals contributes to pedagogical competence as well as mathematical understanding, might well be a subject for an enlarged and a more controlled study."

Abstractor's Notes

As the authors have generally indicated throughout their report, this study is a relatively small scale one and the results need to be viewed in that context. The major implications of the study stem more from the initial rationale for the study than from the statistical results of the analyses which were made. All subjects had previously completed one, and many had completed two three-semester-hour courses in "Mathematics for the Elementary School Teacher." The authors state a continuing need to emphasize content development in mathematics education classes. In the interpretation the authors indicate a concern for exposing their students to the "tools of their future trade." Yet, the test designed to measure the effects of this mathematics education course was centered entirely on mathematics content. The discussion of the study raises interesting questions with which those who teach mathematics education are very familiar. What are the unique objectives and contributions of the content and of the methods courses we prescribe for prospective elementary teachers?

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Section 4-2-000

Listing of Doctoral Theses and Projects*

Summarized in

Dissertation Abstracts International

July thru December 1969**

*Those of potential interest or relevance for mathematics educators and students of mathematics education.

**A starred reference appeared in Dissertation Abstracts prior to July 1969.

4-2-001. Avila, Ramon Luis. A study of the college algebra and trigonometry placement procedures and program for mathematics majors and minors at Ball State University. 30B: 2281-82; November 1969. (The University of Michigan, 1969. Arthur F. Coxford, Jr.) [College/university]

4-2-002. Baker, Truman Dale. A study of the nature of proof in algebraic systems. 30A: 2239; December 1969. (The University of Mississippi, 1969. Jerry Robbins) [Secondary and college/university]

4-2-003. Beougher, Elton Earl. Relationships between success of advanced placement mathematics programs and various administrative, school and community factors. 30A: 195-96; July 1969. (The University of Michigan, 1968. Phillip S. Jones) [Secondary-college/university]

4-2-004. Bidwell, Cecile Gayzik. An experimental study of student achievement in learning per cent as affected by teaching method and teaching pattern. 30A: 919; September 1969. (Temple University, 1968) [Grade 5]

4-2-005. Bierden, James Edward. Provisions for individual differences in seventh grade mathematics based on grouping and behavioral objectives: an exploratory study. 30A: 196-97; July 1969. (The University of Michigan, 1968. Arthur F. Coxford, Jr. and Joseph N. Payne) [Grade 7]

4-2-006. Biggs, Nancy Chisholm. A survey of the mathematics education of West Tennessee elementary school teachers. 30A: 598-99; August 1969. (Memphis State University, 1969. Ford Haynes, Jr.) [In-service elementary]

4-2-007. Bonfield, John Ronald. Predictors of achievement for educable mentally retarded children. 30A: 1009; September 1969. (The Pennsylvania State University, 1968) [MR's, ages 6-7 to 12-6]

4-2-008. Brenton, Beatrice A. An analysis of the effectiveness of a phenomenological approach in teaching, as used in a teacher education modern mathematics program. 30A: 1892; November 1969. (Michigan State University, 1969) [Pre- and in-service]

4-2-009. Buchalter, Barbara Diane Elpern. The validity of mathematics textbook series in grades 7-14 with structure as an objective. 30A: 198-99; July 1969. (University of Arizona, 1968. Raymond Klein and Arthur Steinbrenner) [Grades 7-14]

4-2-010. Bundrick, Charles Michael. A comparison of two methods of teaching selected topics in plane analytic geometry. 30A: 485-86; August 1969. (The Florida State University, 1968. Eugene D. Nichols) [Secondary]

4-2-011. Bursack, Bruce Allen. Utilizing item sampling techniques to scale affective reactions to mathematics. 30A: 1427; October 1969. (The Ohio State University, 1969. Harold C. Trimble) [College/university]

4-2-012. Capper, Victor Lewis. The effects of two types of reinforcement on dropouts, class attendance, and class achievement in a junior-college, continuing-education mathematics program. 30A: 2413-14; December 1969. (Arizona State University, 1969) [Junior college]

4-2-013. Carnes, DuWayne Douglas. A study of the critical thinking ability of secondary summer school mathematics students. 30A: 2242; December 1969. (The University of Mississippi, 1969. Harley F. Garrett) [Secondary]

4-2-014. Ciklamini, Joseph. An investigation to determine if instruction in set concepts enhances the arithmetic problem solving ability of sixth grade children as measured by the Bobbs-Merrill arithmetic achievement test. 30A: 1463; October 1969. (Rutgers-The State University, 1969) [Grade 6]

4-2-015. Clason, Robert Grant. Number concepts in arithmetic texts of the United States from 1880 to 1966, with related psychological and mathematical developments 30A: 146; July 1969. (The University of Michigan, 1968. Phillip S. Jones) [Elementary grades]

4-2-016. Collagan, Robert Bruce. The construction and evaluation of a programmed course in mathematics necessary for success in collegiate physical science. 30A: 1070-71; September 1969. (The Catholic University of America, 1968) [College/university]

4-2-017. Coltharp, Forrest Lee. A comparison of the effectiveness of an abstract and a concrete approach in teaching of integers to sixth grade students. 30A: 923-24; September 1969. (Oklahoma State University, 1968) [Grade 6]

4-2-018. Constantino, Peter Samuel. A study of differences between middle school and junior high school curricula and teacher-pupil classroom behavior. 30A: 614; August 1969. (University of Pittsburgh, 1968) [Grades 7-8]

4-2-019. Coughlin, Sister Francetta. A study of the effectiveness of a modified liberal arts mathematics course in the mathematical preparation of prospective elementary teachers. 30A: 185; July 1969. (The University of Michigan, 1968) [Pre-service elementary]

4-2-020. Craven, Sherralyn Denning. An investigation of methods of teaching a development of the real-number system in college mathematics. 30A: 1072; September 1969. (University of Kansas, 1968) [College/university]

4-2-021. Dansereau, Donald Francis. An information processing model of mental multiplication. 30B: 1916; October 1969. (Carnegie-Mellon University, 1969) [? level of Ss unspecified]

4-2-022. Darnell, Charlotte Deane Potter. Effects of age, ability, and five training procedures on improvement of simple logical thinking of children. 30A: 579; August 1969. (University of Colorado, 1969. Michael Kalk) [Kindergarten and grade 1]

4-2-023. Daugherty, Boice Neal. The influence of the value and size of objects on estimation of their numerousness. 30B: 2434; November 1969. (University of Kentucky, 1965. M. M. White) [College/university]

4-2-024. Deer, George Wendell. The effects of teaching an explicit unit in logic on students' ability to prove theorems in geometry. 30B: 2284-85; November 1969. (The Florida State University, 1969. Eugene D. Nichols) [Secondary]

4-2-025. Donahue, Robert T. An investigation of the factor pattern involved in arithmetic problem solving of eighth grade girls. 30A: 2372; December 1969. (The Catholic University of America, 1969) [Grade 8]

4-2-026. Fairbank, Benjamin Ayer, Jr. Experiments on the temporal aspects of number perception. 30B: 403; July 1969. (University of Arizona, 1969. Neil R. Bartlett) [?]

4-2-027. Forman, Richard William. An analysis of the advanced placement program in mathematics at the University of Illinois and other selected colleges and secondary schools. 30A: 203-204; July 1969. (University of Kansas, 1968) [Secondary-college/university]

4-2-028. Fors, Elton W. Trends and factors in the curriculum choices of the mathematics majors in selected state colleges and universities. 30A: 1895; November 1969. (The University of Oklahoma, 1969. Harold V. Huncke) [College/university]

4-2-029. Foust, William Lee. Enrollment: a curriculum innovation student response measure. 30A: 1465-66; October 1969. (University of Oregon, 1969. John H. Hansen) [Secondary]

4-2-030. Garnett, Emma Whitlock. A study of the relationship between the mathematics knowledge and the mathematics preparation of undergraduate elementary education majors. 30A: 1448; October 1969. (George Peabody College for Teachers, 1968. J. Houston Banks) [Pre-service elementary]

4-2-031. Hagen, David Allan. An analysis of a programmed instructional sequence designed to teach Piaget's mental abilities. 30A: 1888-89; November 1969. (State University of New York at Buffalo, 1969) [Age 11]

4-2-032. Hancox, Frederick James. An investigation of programmed instruction in mathematics as a measure of electronic achievement. 30A: 1913-14; November 1969. (The American University, 1969) [Technical school]

4-2-033. Hand, Charles Reginald. A comparison of permanent pupils and transient military pupils in grades four, five, and six in relation to mathematical mastery. 30A: 207-208; July 1969. (Boston University, 1967. J. F. Weaver) [Grades 4, 5, 6]

4-2-034. Hankins, Donald David, Jr. A fourth grade mathematics program for children from impoverished areas and its effect upon learning. 30A: 2249; December 1969. (United States International University, 1969. Warren W. Hamilton) [Grades 4]

4-2-035. Hanninen, Kenneth Arnold. The effect of texture on tactually perceived length in children. 30A: 169; July 1969. (University of Minnesota, 1968) [Elementary and junior high school]

4-2-036. Hickman, J. D. A study of various factors related to success in first semester calculus. 30A: 2252-53; December 1969. (University of Southern Mississippi, 1969) [College/university]

4-2-037. Hipwood, Stanley James. Pupil growth as a function of teacher flexibility, student independence, and student conformance. 30A: 169-70; July 1969. (The University of New Mexico, 1968) [Junior high school]

4-2-038. Holcombe, Bill Morgan. A study of the relation between student mobility and achievement. 30A: 2253-54; December 1969. (University of South Carolina, 1969) [Grade 9]

4-2-039. Holloway, Regina Hempler. The effect of special in-service training for teachers of the educationally disadvantaged on pupil attitudes and achievement. 30A: 1335; October 1969. (The University of Tulsa, 1969. Elmer F. Ferneau) [Grade 4]

4-2-040. Holmes, Allen Harold. Teaching the logic of statistical analysis by the Monte Carlo approach. 30A: 209; July 1969. (University of Illinois, 1968) [Grade 12]

4-2-041. Horn, Billy Dean. A study of mathematics achievement of selected sixth grade pupils in the public schools of Topeka. 30A: 2254; December 1969. (University of Kansas, 1969) [Grade 6]

4-2-042. Howell, Daisy Loden. The development of an instructional guide for a methods course in secondary school mathematics. 30A: 1050; September 1969. (The University of Mississippi, 1969. Kenneth Russell Bender) [Pre-service secondary]

4-2-043. Jacobsen, Edward Carl. Recommendations for the implementation of a modern mathematics program in Botswana. 30A: 2419-20; December 1969. (University of Kansas, 1969) [Grades K-12]

4-2-044. Jahn, Harvey Raymond. The development of Soviet educational policy during 1917-1936: a case study of mathematics education at the elementary and secondary levels. 30A: 148; July 1969. (The University of Michigan, 1968. W. K. Medlin) [Elementary and secondary]

4-2-045. Jick, Helen. A comparative study of the relative effectiveness of two different freshman mathematics courses. 30B: 1242; September 1969. (Columbia University, 1969. Myron F. Rosskopf) [College/university]

4-2-046. Johnston, Arden Eugene. Audio-tutorial versus traditional instruction in seventh grade mathematics in the Boone junior high school. 30A: 955; September 1969. (Iowa State University, 1969. Richard P. Manatt) [Grade 7]

4-2-047. Juliar, Helen Holt. An analysis of pupil behavior and school marks. 30A: 63; July 1969. (University of Minnesota, 1968. Bruce Balow) [Intermediate grades]

4-2-048. Kansky, Robert James. An analysis of models used in Australia, Canada, Europe, and the United States to provide an understanding of addition and multiplication over the natural numbers. 30A: 1074-75; September 1969. (University of Illinois, 1969) [Elementary grades]

4-2-049. Kavett, Phyllis Flyer. A study of the teaching of non-decimal systems of numeration in the elementary school. 30A: 1468; October 1969. (Rutgers-The State University, 1969) [Grades 4 and 6]

4-2-050. Keane, Daniel Ford. Relationships among teacher attitude, student attitude, and student achievement in elementary school arithmetic. 30A: 64-65; July 1969. (The University of Florida, 1968. Maurice Ahrens) [Intermediate grades]

4-2-051. Kester, Scott Woodrow. The communication of teacher expectations and their effects on the achievement and attitudes of secondary school pupils. 30A: 1434-35; October 1969. (The University of Oklahoma, 1969. Henry L. Angelino) [Grade 7]

4-2-052. Kleckner, Lester Gerald. An experimental study of discovery type teaching strategies with low achievers in Basic Mathematics I. 30A: 1075-76; September 1969. (The Pennsylvania State University, 1968) [Grades 9 and 10]

4-2-053. Kneitz, Margaret H. A study of secondary mathematics teacher drop-outs in Texas. 30A: 2402; December 1969. (University of Houston, 1969) [Secondary in-service]

4-2-054. Knight, Carlton Walter. The impact of atmospheric pressure upon mathematical performance. 30A: 495; August 1969. (University of Oregon, 1968. A. A. Sandin) [Grade 5]

4-2-055. Lamon, William Emile. Structural learning characteristics of children from differing instructional backgrounds as manifested in the learning of certain mathematical groups. 30A: 212-13; July 1969. (University of California, Berkeley, 1968. Arden K. Ruddell and Robert M. Gagne) [Grades 4, 5, 6]

4-2-056. Lovett, Carl James. An analysis of the relationship of several variables to achievement in first year algebra. 30A: 1470; October 1969. (The University of Texas at Austin, 1969. Ralph W. Cain) [Secondary grades]

4-2-057. Mallory, Curtiss Orville. An experiment using programmed materials to supplement a mathematics content course for elementary education majors. 30B: 1793; October 1969. (Colorado State College, 1969. [College/university, pre-service elementary]

4-2-058. Mastbaum. Sol. A study of the relative effectiveness of electric calculators or computational skills kits in the teaching of mathematics. 30A: 2422-23; December 1969. (University of Minnesota, 1969) [Grades 7 and 8]

4-2-059. Mazur, Joseph Lawrence. Validity of scholastic aptitude scores as predictors of achievement. 30A: 171; July 1969. (Case Western Reserve University, 1968) [Elementary grades]

4-2-060. McAloon, Sister Mary deLourdes. An exploratory study of teaching units in logic to grades three and six. 30A: 1918-19; November 1969. (The University of Michigan, 1969. Joseph N. Payne) [Grades 3 and 6]

4-2-061. McKinney, Eugene Barton. A study of high school honors classes in English and mathematics and academic success in college. 30A: 1369; October 1969. (St. Louis University, 1968) [Secondary and college/university]

4-2-062. Michael, Elizabeth Marion Bodi. The effect of human relations training upon the academic achievement of pre- and early adolescent children. 30A: 1340; October 1969. (Wayne State University, 1968. Morrel J. Clute) [Grades 4-6]

4-2-063. Morgan, Richard Thomas. The role of the digital computer in a general education course in mathematics. 30A: 71-72; July 1969. (Columbia University, 1968. Bruce R. Bogeli) [? college/university ?]

4-2-064. Naramore, Vincent H. Cognitive continuity: a study of the secondary school teachers' knowledge of the field properties of mathematical systems. 30A: 191-92; July 1969. (Syracuse University, 1968) [In-service secondary]

4-2-065. Nelson, Bruce Edward. The relationship of mathematics in Oregon high schools to placement and success in first-year mathematics at Oregon State University. 30A: 1401-1402; October 1969. (Oregon State University, 1969. Howard L. Wilson) [Secondary-college/university]

4-2-066. Nelson, Leonard Theodore. The relationship between verbal, visual-spatial, and numerical abilities and the learning of the mathematical concept of function. 30A: 218; July 1969. (The University of Michigan, 1968. Joseph N. Payne) [Grades 8]

4-2-067. Nelson, Paul Alden. Attitudes held by elementary education teachers toward the developmental potential of the content areas. 30A: 192; July 1969. (University of Illinois, 1968) [Pre- and in-service elementary]

4-2-068. Niman, John. Mathematical models of physics for teaching. 30A: 2264; December 1969. (Columbia University, 1969. Paul C. Rosenbloom) [College/university]

4-2-069. Norris, Fletcher Ragland. Pupil achievement as a function of an in-service training program on mathematics concepts for sixth grade teachers. 30A: 1054; September 1969. (George Peabody College for Teachers, 1968. Otto C. Bassler) [In-service elementary; grade 6]

4-2-070. Nowak, Stanley Marion. The development and analysis of the effects of an instructional program based on Piaget's theory of classification. 30A: 1875; November 1969. (State University of New York at Buffalo, 1969) [Grades 1-3]

4-2-071. Nugent, Paul Thomas. A study of selected elementary teachers' attitudes toward the new mathematics. 30A: 2265; December 1969. (University of Kentucky, 1969. Martha Sudduth and Wallace Ramsey) [In-service elementary and junior high school]

4-2-072. O'Daffer, Phares Glyn. An exploratory study of the abilities of fifth and seventh grade mathematics students to learn finite group properties and structures. 30A: 219; July 1969. (University of Illinois, 1968) [Grades 5 and 7]

4-2-073. Olsen, Clarence Randall. The effects of enrichment tutoring upon self-concept, educational achievement, and measured intelligence of male underachievers in an inner-city elementary school. 30A: 2404; December 1969. (Michigan State University, 1969) [Grades 2-4]

4-2-074. Pang, Paul Hau-lim. A mathematical and pedagogical study of square root extraction. 30A: 1080; September 1969. (State University of New York at Buffalo, 1969) [Grades 8 and 9]

4-2-075. Partner, Bruce Earl. A comparison of achievement of main and branch campus mathematics students. 30A: 72; July 1969. (The Ohio State University, 1968. Harold C. Trimble) [College/university]

4-2-076. Perrin, Jerry Dale. A statistical and time change analysis of achievement differences of children in a nongraded and a graded program in selected schools in the Little Rock public schools. 30A: 530-31; August 1969. (University of Arkansas, 1969. R. M. Roelfs) [Elementary]

4-2-077. Pfaeffle, Heinz. A comparison of two educational programs for beginning instruction with educable mentally retarded pupils. 30A: 174-75; July 1969. (The University of Wisconsin-Madison, 1968. James J. McCarthy) [MR's, ages 7-6 to 10-1]

4-2-078. Pruitt, Ralph Lewis. An analysis of types of exercises in plane geometry texts in the United States from 1878 to 1966. 30A: 1414; October 1969. (The Ohio State University, 1969. Nathan Lazar) [Secondary]

4-2-079. Reimer, Dennis D. The effectiveness of a guided discovery method of teaching in a college mathematics course for non-mathematics and non-science majors. 30A: 626; August 1969. (North Texas State University, 1969) [College/university]

*4-2-080. Richards, Hyrum Edwards. Variations on Piaget's pre-number developmental tests used as learning experiences. 29B: 3114; February 1969. (Utah State University, 1968. David R. Stone) [MR's, age ?]

4-2-081. Roberge, James Joseph. An investigation of children's abilities to reason with principles of class reasoning and their isomorphs in conditional reasoning. 30A: 592-93; August 1969. (The University of Connecticut, 1969) [Grades 4-10]

4-2-082. Robitaille, David Ford. Selected behaviors and attributes of effective mathematics teachers. 30A: 1472-73; October 1969. (The Ohio State University, 1969. Harold C. Trimble) [Secondary in-service]

4-2-083. Rockhill, Theron David. Programmed instruction vs. problem session as the supplement to large group instruction in college mathematics. 30B: 2305-2306; November 1969. (State University of New York at Buffalo, 1969. Harriet F. Montague) [College/university]

4-2-084. Sams, Orval J. The ability to conserve volume of a solid among selected Indian and Caucasian pupils. 30A: 1344; October 1969. (University of Arizona, 1969. Milo K. Blecha) [Grades 5 and 6]

4-2-085. Sanders, Beverly Jean. Experimental analysis of visual matching-to-sample in children. 30B: 418; July 1969. (McGill University, 1969) [Kindergarten]

4-2-086. Scannicchio, Thomas Henry. Student achievement in college calculus, Louisiana State University 1967-1968. 30A: 1344-45; October 1969. (Louisiana State University and Agricultural and Mechanical College, 1969. Sam Adams) [College/university]

4-2-087. Schilling, Frank Charles. A description of the development and implementation of a curriculum-materials package for teaching mathematics to low achievers. 30A: 1925-26; November 1969. (University of Pittsburgh, 1969) [Grade 4]

4-2-088. Sharp, Richard Malcolm. An experimental study of two approaches to teaching informal geometry in grades three and four. 30A: 74-75; July 1969. (Boston University, 1967. J. F. Weaver) [Grades 3 and 4]

4-2-089. Sinks, Thomas Alonzo. How individualized instruction in junior high science, mathematics, language arts, and social studies affects student achievement. 30A: 224-25; July 1969. (University of Illinois, 1968) [Grade 7]

4-2-090. Smith, Charles Winston. A study of constant errors in subtraction in the application of selected principles of the decimal numeration system made by third and fourth grade students. 30A: 1084; September 1969. Wayne State University, 1968. Charlotte W. Junge) [Grades 3 and 4]

4-2-091. Sneed, Billy Ray. A study of the qualifications of mathematics instructors in the regionally accredited public junior colleges of Mississippi. 30A: 2407; December 1969. (The University of Mississippi, 1969. John R. Fawcett, Jr.) [Junior college teachers]

4-2-092. Sollee, Natalie Dosick. Verbal competence and the acquisition of conservation. 30B: 2409-10; November 1969. (Boston University, 1969. Chester C. Bennett) [Grades 1 and 2]

4-2-093. Steinberg, Seymour. An investigation of the effects of vitamin B_{12} on mathematics learning. 30A: 75-76; July 1969. (New York University, 1968.) [College/university]

4-2-094. Stumpff. Howard Keith. The nature and levels of rigor in the teaching of college calculus. 30A: 226; July 1969. (University of Kansas, 1968) [College/university]

4-2-095. Sweetser, Evan Alton. A descriptive case study of an elementary teacher education program of science, mathematics, and reading for experienced teachers. 30A: 2409; December 1969. (Michigan State University, 1969) [In-service elementary]

4-2-096. Taylor, Ralph Clinton. An investigation of the value of drill on arithmetic fundamentals by use of tape recordings which combine techniques developed in language laboratories with principles of programmed learning. 30A: 1928. (University of Southern California, 1969. Finn) [Grade 5]

4-2-097. Tener, Morton. Teaching business mathematics by differentiated methodologies. 30A: 1087; September 1969. (Temple University, 1968) [1 secondary]

4-2-098. Tietz, Naunda Meier. A comparison of two methods of teaching multiplication: repeated-addition and ratio-to-one. 30A: 1060; September 1969. (Oklahoma State University, 1968) [Grade 4]

4-2-099. Tismer, Werner Dietrich. The relationship between organizational structure and the academic achievement and school adjustment of sixth grade pupils. 30A: 2314; December 1969. (University of Minnesota, 1969. Carl Goosen and Clifford Hooker) [Grade 6]

4-2-100. Treffinger, Donald John. The effects of programmed instruction in productive thinking on verbal creativity and problem solving among pupils in grades four, five, six, and seven. 30A: 1031; Setpember 1969. (Cornell University, 1969) [Grades 4-7]

4-2-101. Vickrey, Thomas Loren. Fibonacci numbers. 30A: 939; September 1969. (Oklahoma State University, 1968) [Secondary and college/university]

4-2-102. Wade, Harmon V. A case study of the role of a superintendent of schools in district-wide curriculum development in modern mathematics and science education and an evaluation of resulting educational outcomes. 30A: 119-20; July 1969. (New York University, 1968. Richard C. Lonsdale) [Grades K-12]

4-2-103. Wagman, Harriett Gordon. A study of the child's conception of area measure. 30A: 1350; October 1969. (Columbia University, 1968. Myron F. Rosskopf) [Ages 8, 10, 11]

4-2-104. Walken, Christine Louise. Accelerating classroom attending behaviors and learning rate. 30A: 1032; September 1969. (University of Oregon, 1968. Hill M. Walker) [Grade 4]

4-2-105. Werner, Donald C. An educational analysis of certain philosophical implications in modern mathematics. 30A: 1350-51; October 1969. (The Catholic University of America, 1968)

4-2-106. Weston, Leslie Donald. An exploration of the interrelationships among children's arithmetic achievement, their styles of learning, their responsibility for intellectual academic achievement, and their parents' attitudes. 30A: 1087-88; September 1969. (Wayne State University, 1968. Charlotte W. Junge) [Grades 4 and 6]

4-2-107. William, Kenneth Edwin. The feasibility of the use of dimensional and unit analysis in junior high school science instruction. 30A: 77; July 1969. (Indiana University, 1968. Ronald C. Welch) [Grade 8]

4-2-108. Willson, George Hayden. A comparison of decimal-common fraction sequence with conventional sequence for fifth grade arithmetic. 30A: 1762; November 1969. (University of Arizona, 1969. Milo K. Blecha) [Grade 5]

4-2-109. Wolfe, Richard Edgar. Strategies of justification used in the classroom by teachers of secondary school mathematics. 30A: 1064-65; September 1969. [In-service secondary] (University of Illinois, 1969)

4-2-110. Young, James Clark. An evaluation of a pontoon transitional design--ninth grade low ability level students. 30A: 1931; November 1969. (University of Southern California, 1969. Georgiades) [Grade 9]